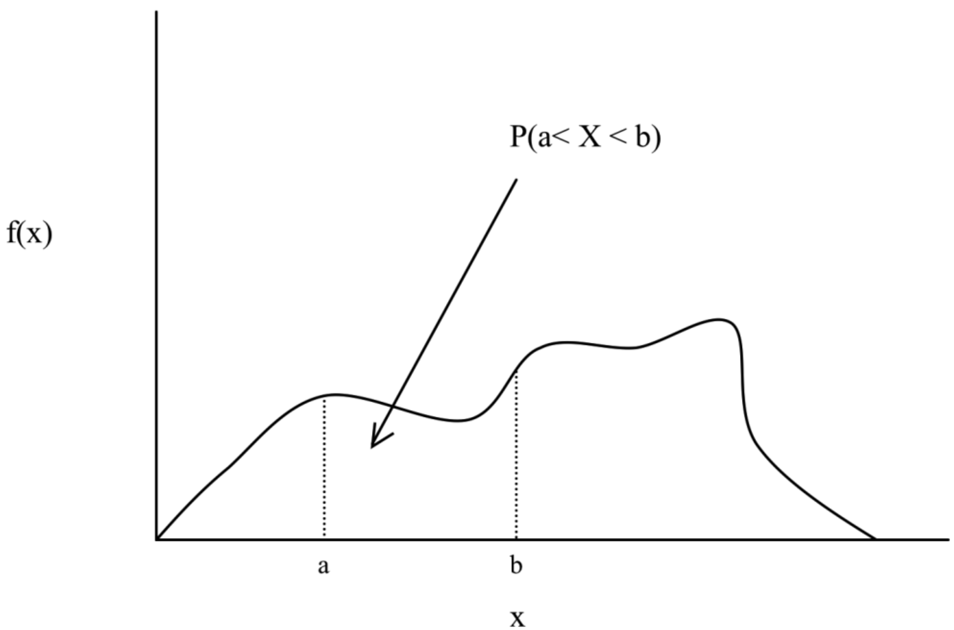
CIVL 7012/8012

Continuous Distributions

Probability Density Function



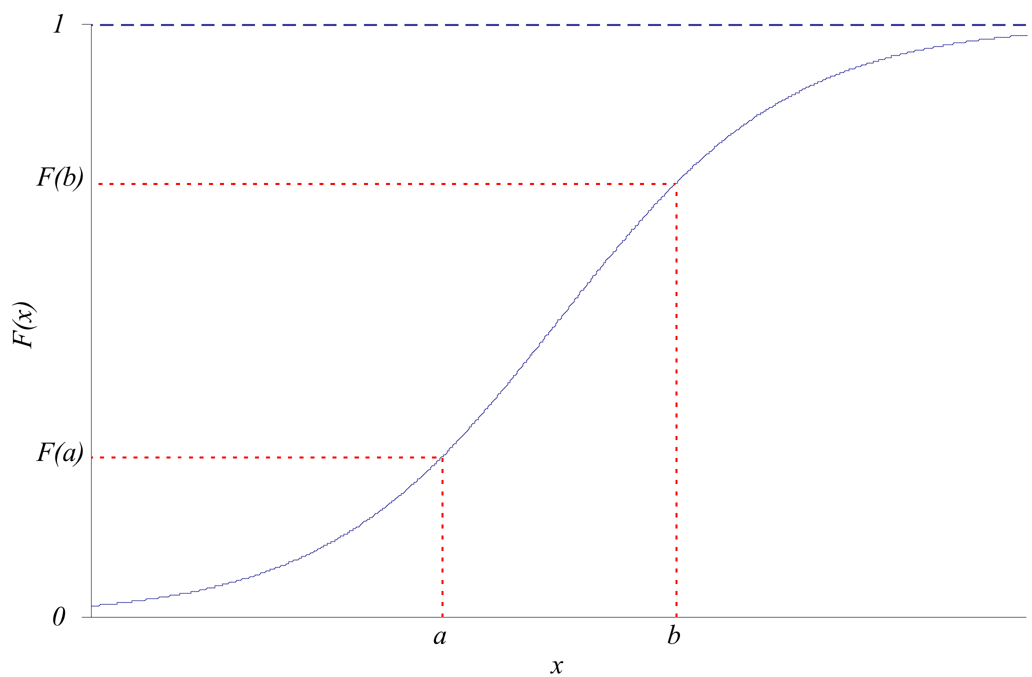
$$P(a \le X \le b) = \int_a^b f(x)dx$$

Probability Density Function

- Definition:
  - $f(x) \geq 0$
  - $\int_{-\infty}^{\infty} f(x) = 1$ and,
  - $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$.

Cumulative Distribution Function

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$



$$P(X \le a) = F(a)$$

Example

Suppose the cumulative distribution function of the random variable X is:

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \le x < 5 \\ 1 & x \ge 5 \end{cases}$$

Determine: The probability density function of x and P(x < 2.8).

Continuous Distributions

- The probability that the random variable X will take on a range of values is: $P(a \le X \le b) = F(b) - F(a)$
- Expected Value:

$$E(x) = \mu_x = \int_{-\infty}^{\infty} x f(x)\,dx$$

- Variance:

$$V(x) = \sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)\,dx = E(X^2) - \left[E(X)\right]^2$$

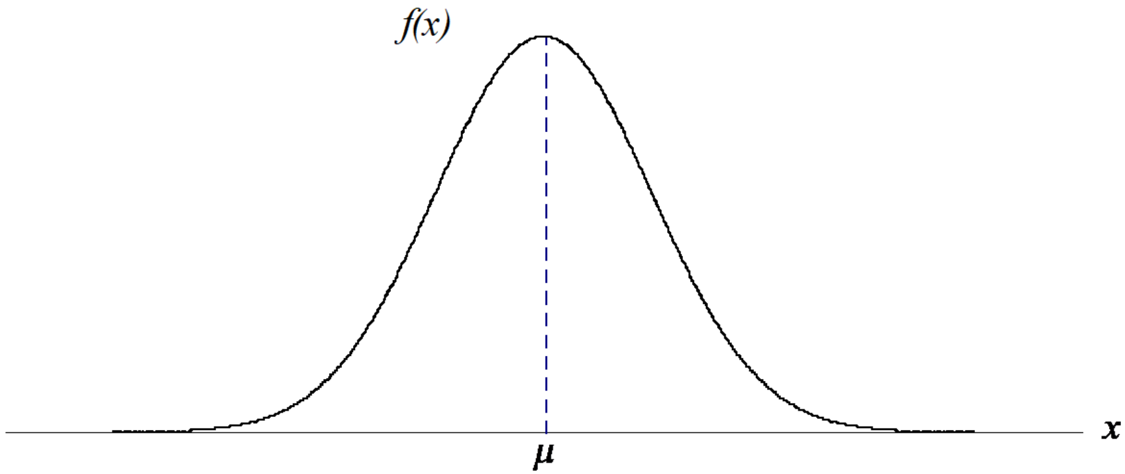
Important Continuous Distributions

- Normal Distribution
- Exponential Distribution
- Gamma Distribution
- Weibull Distribution
- Lognormal Distribution

The Normal Distribution

The normal distribution is the foundation of many statistical methods used in data analysis because it *does* accurately describe the distribution of ***random errors***. Its importance cannot be overstated.

The normal distribution is bell-shaped, symmetrical about the mean, μ, and ranges from $-\infty$ to ∞.



The Normal Distribution

The probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \begin{array}{c} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{array}$$

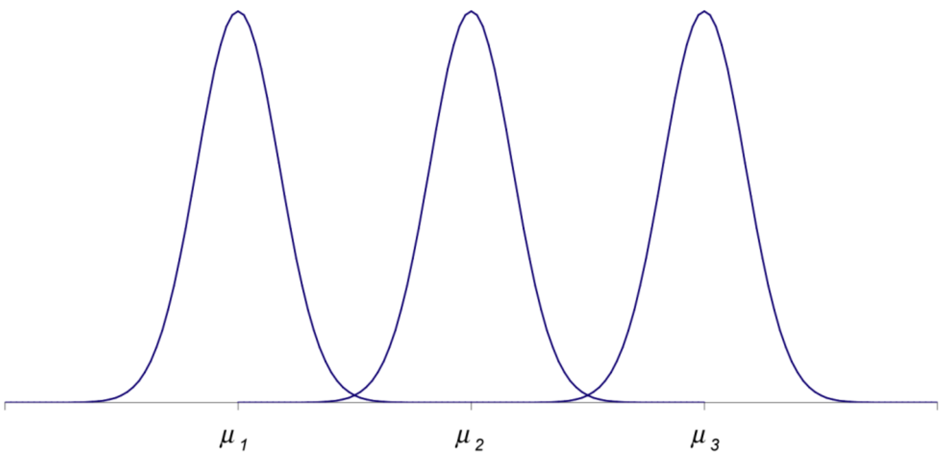
We have a shorthand way of describing a normally distributed random variable:

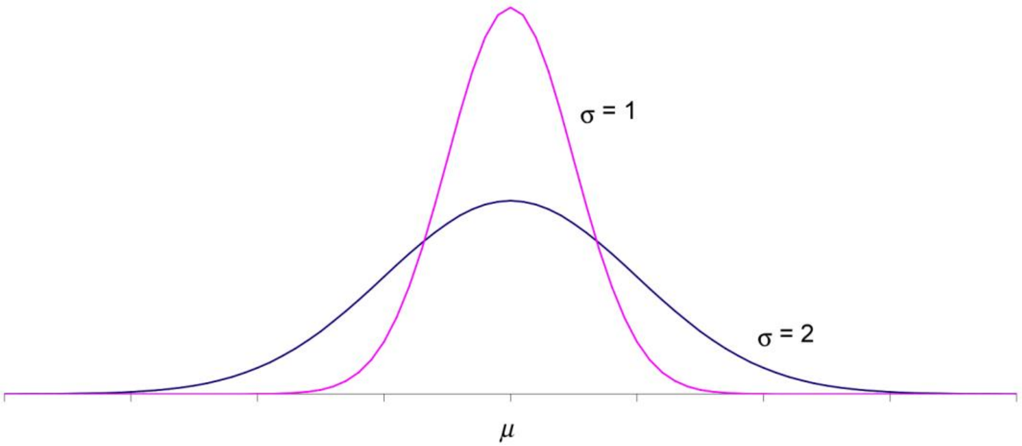
$$X \sim \mathrm{N}[\mu, \sigma^2]$$

This says "the random variable X is normally distributed with mean μ and variance σ^2."

The cumulative distribution function is given by $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2} du$

The Normal Distribution





The Standard Normal Distribution

If X is a normally distributed random variable with mean μ and variance σ^2, then $(X-\mu)/\sigma$ is a normally distributed random variable with zero mean and unit variance. In "shorthand" notation:

$$\text{If } X \sim N\left[\mu,\sigma^2\right] \text{ then } \frac{X-\mu}{\sigma} \sim \mathrm{N}[0,1]$$

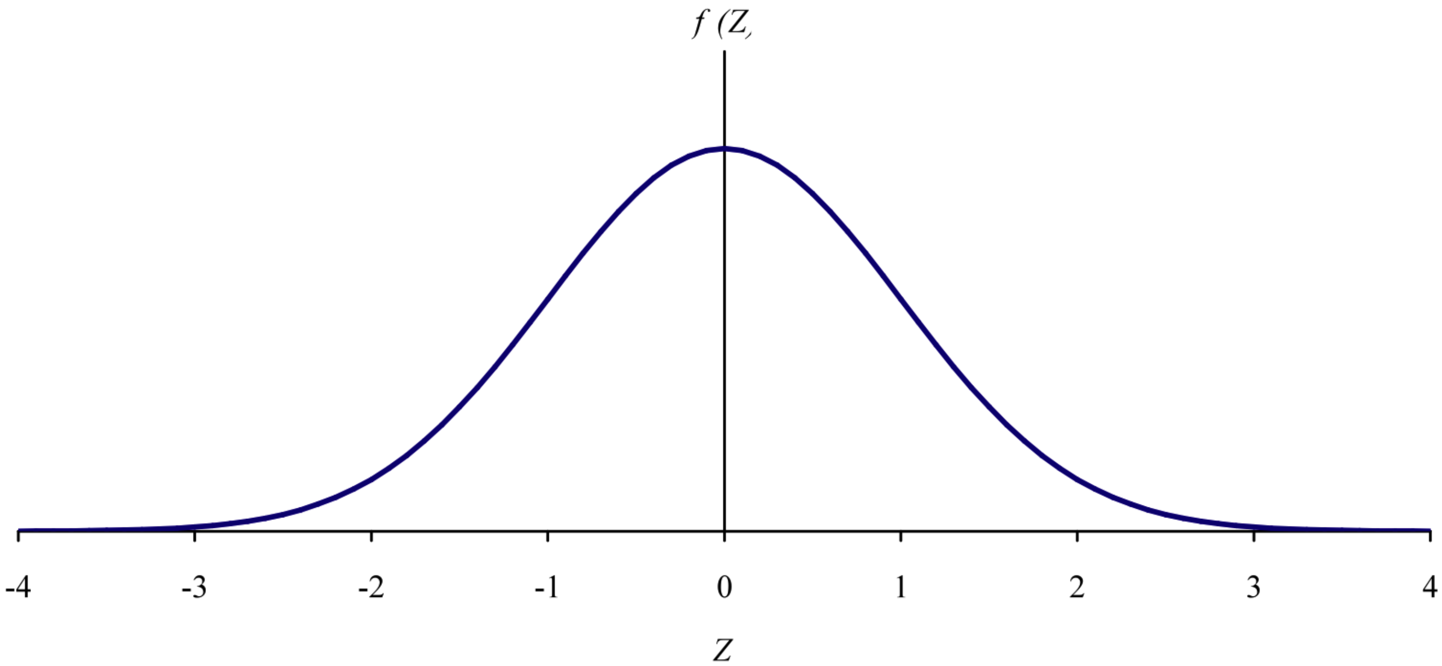
For convenience, we define the Z statistic as:

$$Z = \frac{X-\mu}{\sigma}$$

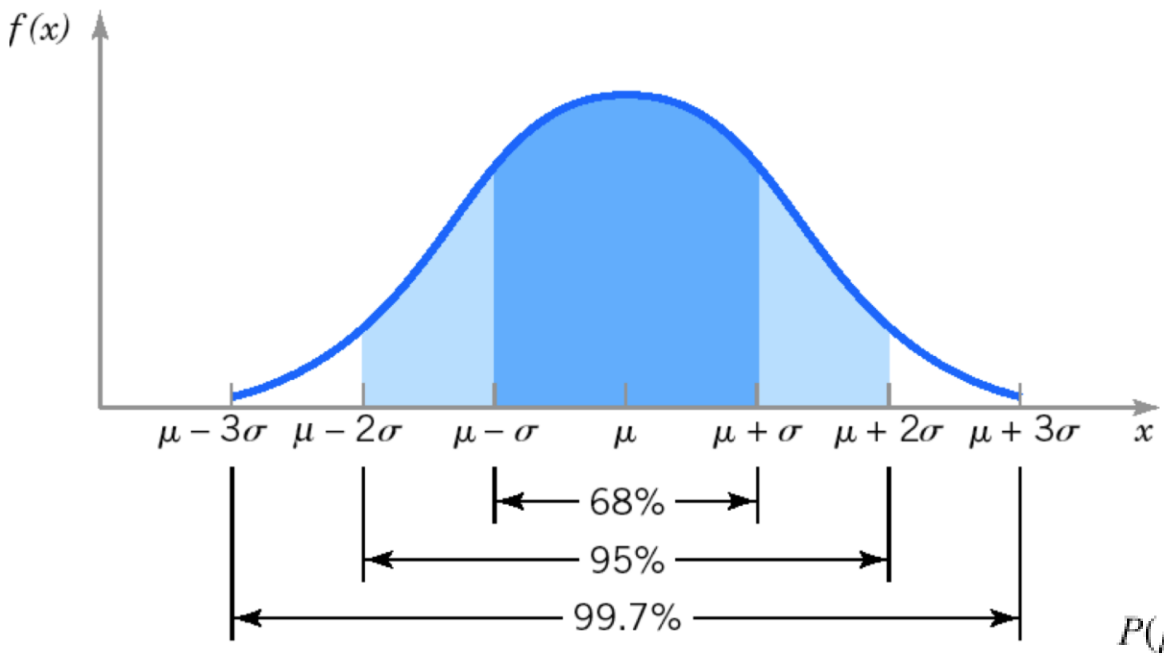
so we can write

$$Z \sim \mathrm{N}[0,1]$$

The Standard Normal Distribution



Standard Normal Distribution

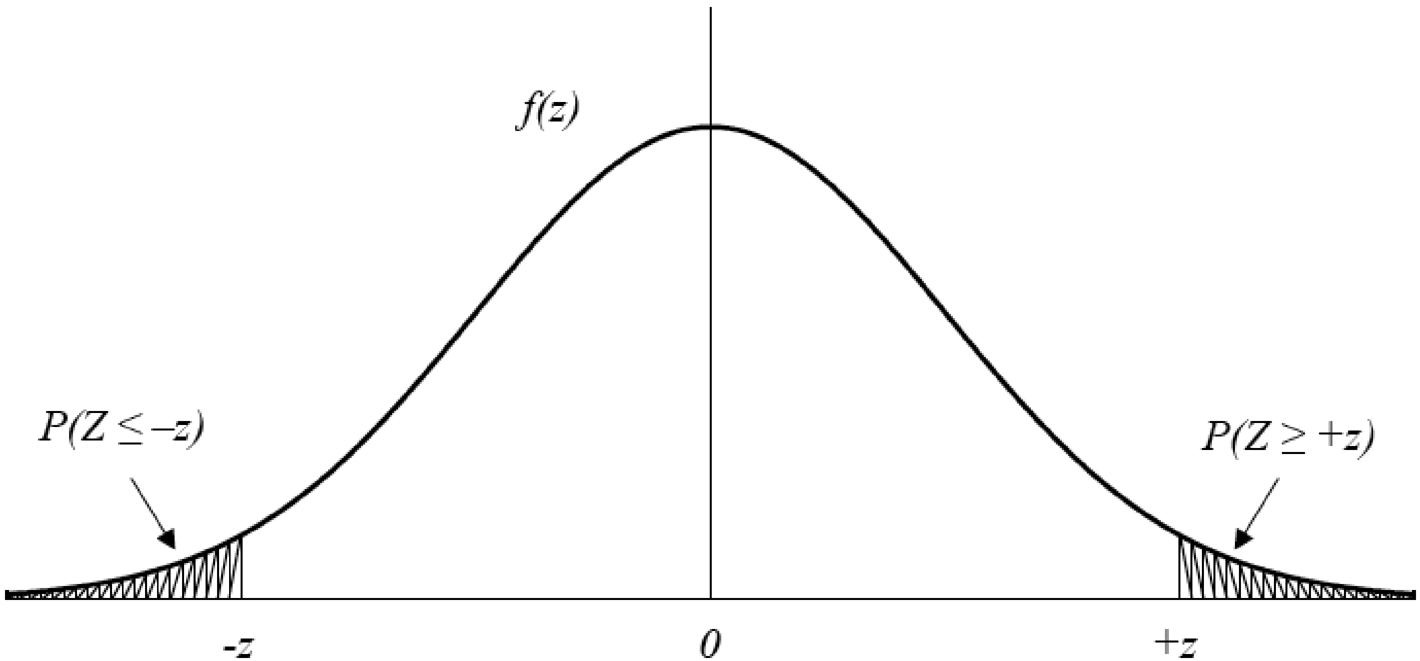


$$P(\mu-\sigma < X < \mu+\sigma) = 0.6827$$

$$P(\mu-2\sigma < X < \mu+2\sigma) = 0.9545$$

$$P(\mu-3\sigma < X < \mu+3\sigma) = 0.9973$$

The Standard Normal Distribution

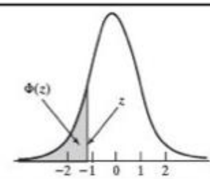


$$F(z) = P(Z \le z)$$

$$F(-z) = 1 - F(+z)$$

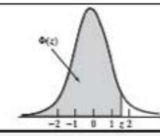
Table C.4 Standard Normal Distribution Function (c.d.f.)

$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$$



Normal Distribution Table (Each entry is the total area under the standard normal curve to the left of z, which is specified to two decimal places by joining the row value to the column value.)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
−3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
−3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1921	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

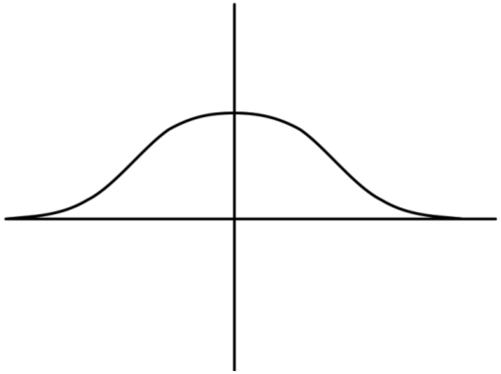
Selected Upper Percentage Points

Tail probability, α	0.100	0.050	0.025	0.010	0.005
Upper percentage point, $z(\alpha)$	1.282	1.645	1.960	2.326	2.576

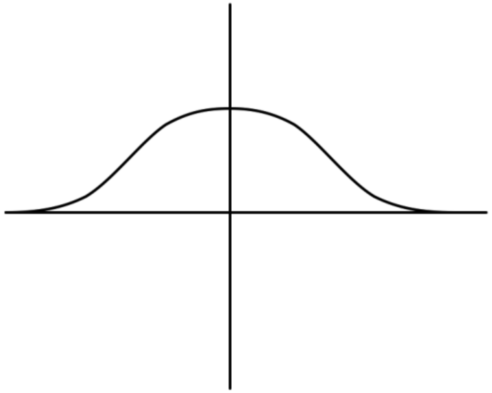
Source: Reproduced in abridged form from Table 1 of E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians,* Vol. 1 (Cambridge: Cambridge University Press, 1954).

Example:

1. P(Z < 2.55)

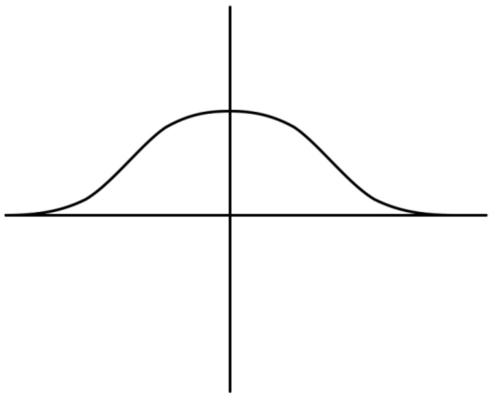


1. P(Z > 1.26)

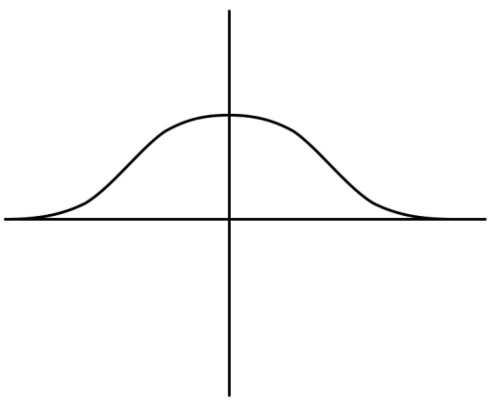


Example:

1. $P(Z > -1.37)$



1. $P(-1.25 < Z < 0.37)$



Example:

Example:

In diaphragms of rats, tissue respiration rate under standard temperature conditions is normally distributed with $\mu = 2.03$ and $\sigma = 0.44$.

a. What is the probability that a randomly selected rat has rate $X>2.5$?

b. What is the probability that X falls outside the interval (1.59, 2.47)?

Example:

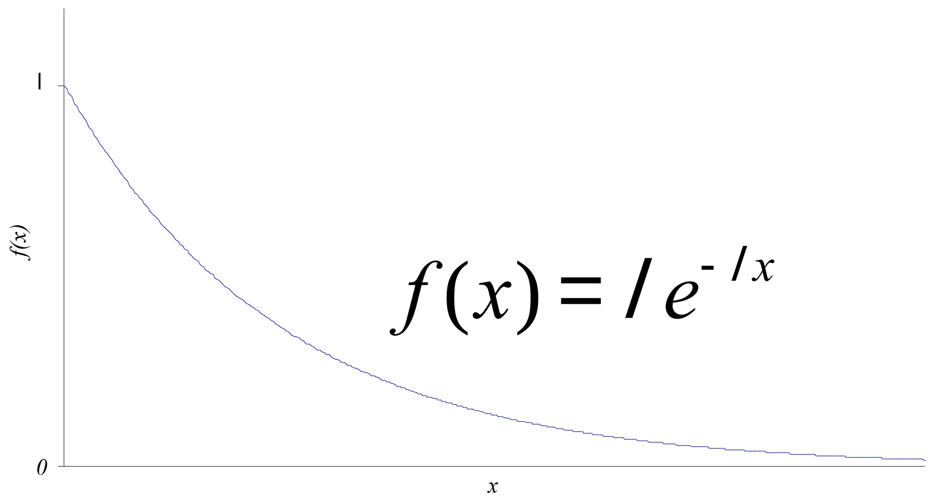
In an industrial process, the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be 3.0 $\pm$ 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean 3.0 and standard deviation 0.005. On the average, how many manufactured ball bearings will be scrapped?

Example:

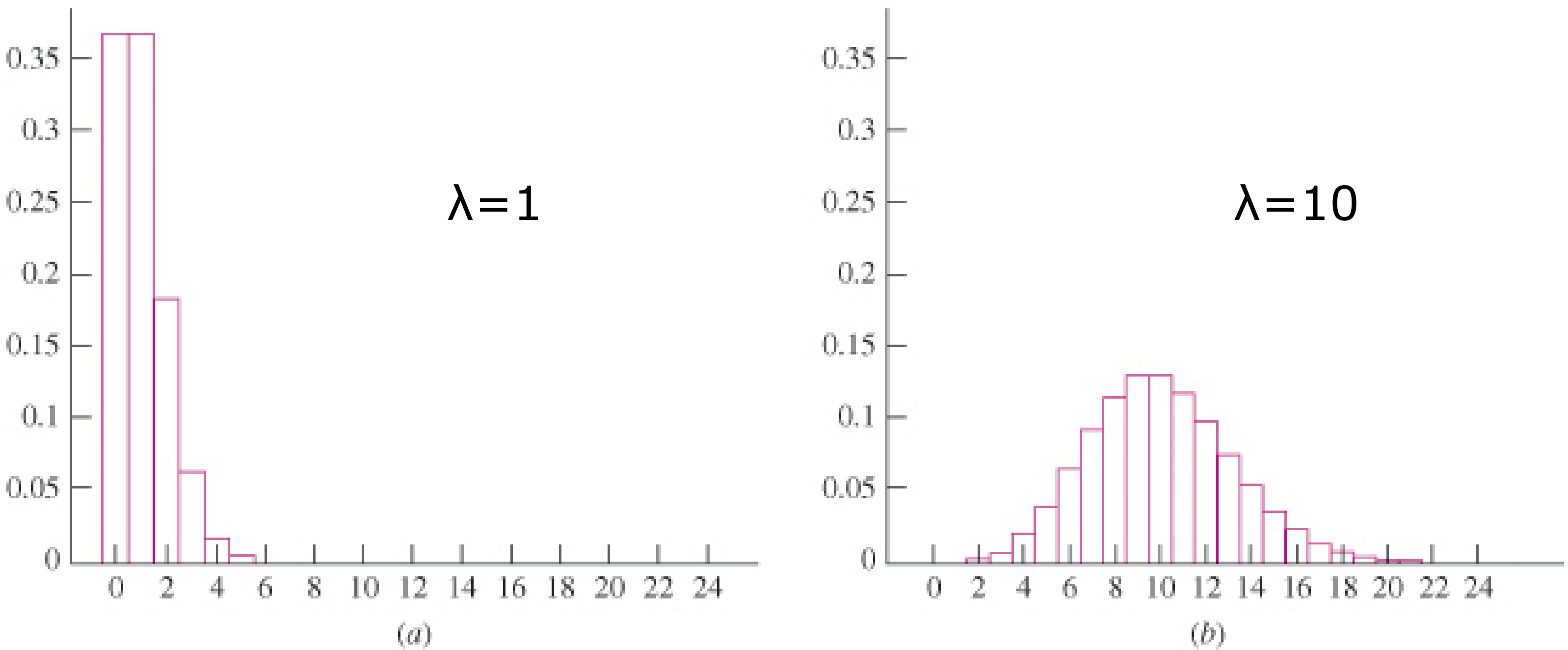
Gauges are used to reject all components where a certain dimension is not within the specification $1.5 \pm d$. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.2 Determine the value d such that the specifications "cover" 95% of the measurements.

The Exponential Distribution

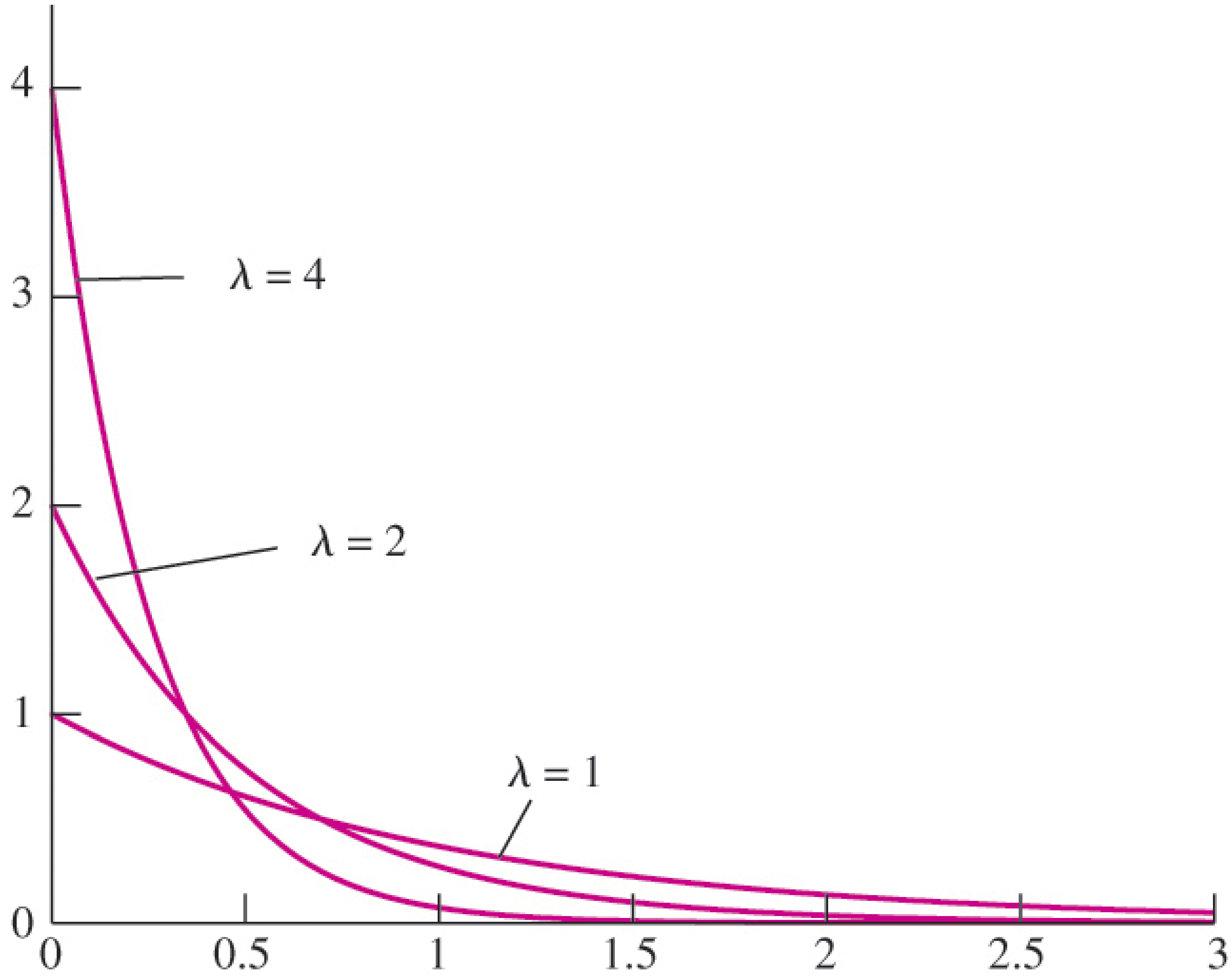
- Frequently used to model time between successive events (arrivals or failures).
- Models the continuous "unit" versus the discrete event (Poisson).



The Poisson Distribution



The Exponential Distribution



The Exponential Distribution

- Cumulative Distribution Function:

$$F(x) = 1 - e^{- / x}$$

- Expected Value:

$$E(x) = m = \frac{1}{/}$$

- Variance:

$$V(x) = s^2 = \frac{1}{/^2}$$

Example: The Exponential Distribution

At a stop sign location on a cross street, vehicles require headways of 6 seconds or more in the main street traffic before being able to cross. If the total flow rate of the main street traffic is 1200 vph, what is the probability that any given headway will be greater than 6 seconds?

Gamma Distribution

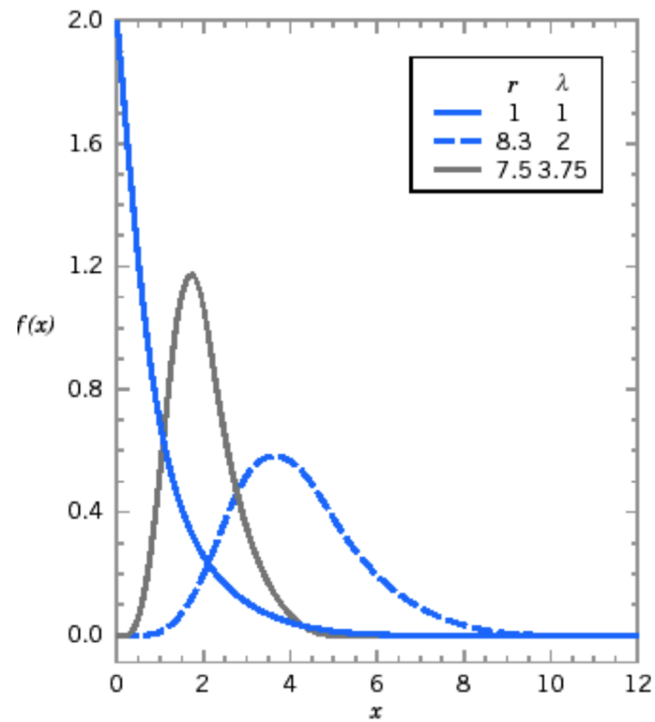
$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}$$

$$Gamma\ Function: \quad \Gamma(r) = \int_0^\infty x^{r-1} e^{-x}\, dx$$

$$E(x) = \mu = \frac{r}{\lambda}$$

$$\sigma^2 = v(x) = \frac{r}{\lambda^2}$$

$$F(x) = P(T \le x) = \begin{cases} 1 - \sum_{j=0}^{r-1} e^{-\lambda x} \frac{(\lambda x)^j}{j!} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$$



Gamma Function

Properties of the Gamma function:

- $\Gamma(1) = 1$
- For n>1:
- $\Gamma(n) = (n-1)\,\Gamma(n-1)$
- $\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$
- $\Gamma(n+1) = n!$

*** GAMMA FUNCTION**

Values of $\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx;\ \Gamma(n+1) = n\Gamma(n)$

n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$	n	$\Gamma(n)$
1.00	1.00000	1.25	.90640	1.50	.88623	1.75	.91906
1.01	.99433	1.26	.90440	1.51	.88659	1.76	.92137
1.02	.98884	1.27	.90250	1.52	.88704	1.77	.92376
1.03	.98355	1.28	.90072	1.53	.88757	1.78	.92623
1.04	.97844	1.29	.89904	1.54	.88818	1.79	.92877
1.05	.97350	1.30	.89747	1.55	.88887	1.80	.93138
1.06	.96874	1.31	.89600	1.56	.88964	1.81	.93408
1.07	.96415	1.32	.89464	1.57	.89049	1.82	.93685
1.08	.95973	1.33	.89338	1.58	.89142	1.83	.93969
1.09	.95546	1.34	.89222	1.59	.89243	1.84	.94261
1.10	.95135	1.35	.89115	1.60	.89352	1.85	.94561
1.11	.94739	1.36	.89018	1.61	.89468	1.86	.94869
1.12	.94359	1.37	.88931	1.62	.89592	1.87	.95184
1.13	.93993	1.38	.88854	1.63	.89724	1.88	.95507
1.14	.93642	1.39	.88785	1.64	.89864	1.89	.95838
1.15	.93304	1.40	.88726	1.65	.90012	1.90	.96177
1.16	.92980	1.41	.88676	1.66	.90167	1.91	.96523
1.17	.92670	1.42	.88636	1.67	.90330	1.92	.96878
1.18	.92373	1.43	.88604	1.68	.90500	1.93	.97240
1.19	.92088	1.44	.88580	1.69	.90678	1.94	.97610
1.20	.91817	1.45	.88565	1.70	.90864	1.95	.97988
1.21	.91558	1.46	.88560	1.71	.91057	1.96	.98374
1.22	.91311	1.47	.88563	1.72	.91258	1.97	.98768
1.23	.91075	1.48	.88575	1.73	.91466	1.98	.99171
1.24	.90852	1.49	.88595	1.74	.91683	1.99	.99581
						2.00	1.00000

* For large positive values of x, $\Gamma(x)$ approximates Stirling's asymptotic series

$$x^x e^{-x}\sqrt{\frac{2\pi}{x}}\left[1 + \frac{1}{12x} + \frac{1}{288x^2} - \frac{139}{51840x^3} - \frac{571}{2488320x^4} + \cdots\right].$$

Example

Suppose the survival time, in weeks, of a randomly selected male mouse exposed to 240 rads of gamma radiation has a gamma distribution with $r = 8$ and $\lambda = 1/15$. Find:

a.) The expected survival time;

b.) variance;

c.) the probability that a mouse survives between 60 and 120 weeks;

d.) the probability that a mouse survives at least 30 weeks.

Weibull Distribution

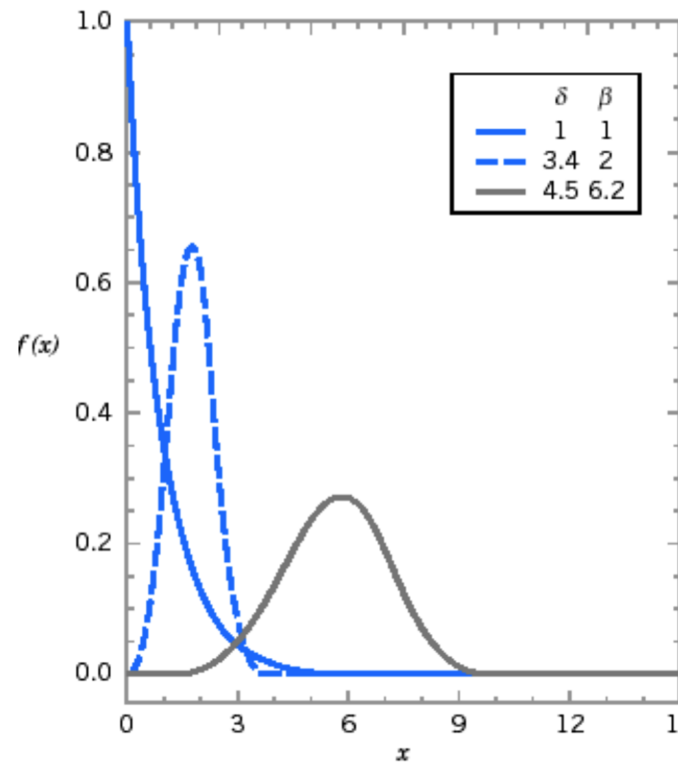
$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} e^{-\left(\frac{x}{\delta}\right)^{\beta}} \quad for \quad x > 0,$$

where δ = *scale parameter and* β = *shape parameter.*

$$F(x) = 1 - e^{-\left(\frac{x}{\delta}\right)^{\beta}}$$

$$E(x) = \mu = \delta\,\Gamma\left(1 + \frac{1}{\beta}\right)$$

$$V(x) = \sigma^2 = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2$$



Example

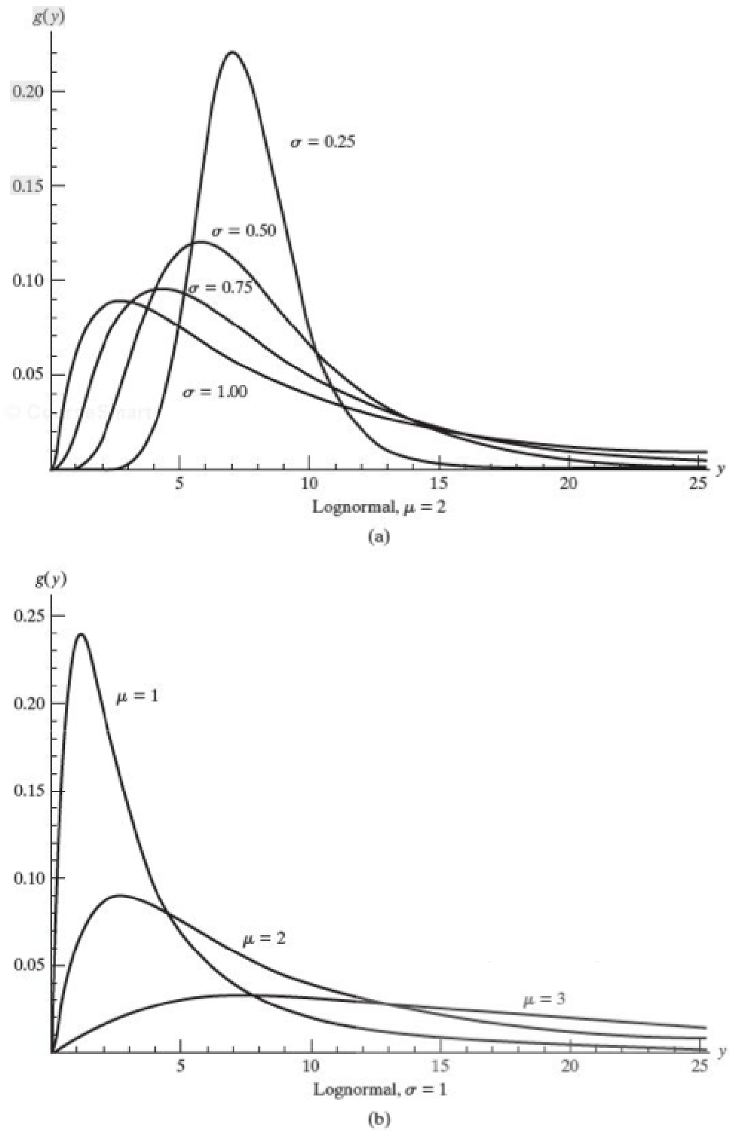
Researchers suggest using a Weibull distribution to model the duration of a bake step in the manufacture of a semiconductor. Let T represent the duration in hours of the bake step for a randomly chosen lot. If T follows a Weibull distribution having $\beta=0.3$, and $\delta=10$, what is the probability that the bake step takes longer than four hours? What is the probability that it takes between two and seven hours?

Lognormal Distribution

$$\mathcal{N}(\ln x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right], \quad x > 0.$$

$$\mu = \ln(\mathrm{E}[X]) - \frac{1}{2}\ln\left(1 + \frac{\mathrm{Var}[X]}{(\mathrm{E}[X])^2}\right) = \ln(\mathrm{E}[X]) - \frac{1}{2}\sigma^2,$$

$$\sigma^2 = \ln\left(1 + \frac{\mathrm{Var}[X]}{(\mathrm{E}[X])^2}\right).$$


g(y)
0.20
0.15
0.10
0.05
σ = 0.25
σ = 0.50
σ = 0.75
σ = 1.00
5
10
15
20
25
y
Lognormal, μ = 2
(a)
g(y)
0.25
0.20
0.15
0.10
0.05
μ = 1
μ = 2
μ = 3
5
10
15
20
25
y
Lognormal, σ = 1
(b)

Example

The time between sever earthquakes at a given region follows a lognormal distribution with a coefficient of variation of 40%. The expected time between severe earthquakes is 80 yrs.

a.) Determine the parameters of this lognormally distributed recurrence time.

b.) Determine the probability that a severe earthquake will occur within 20 yr from the previous one.

c.) Suppose the last severe earthquake in the region took place 100 yrs ago. What is the probability that a severe earthquake will occur over the next year?

Table 3.3-1 Important Continuous Distributions

Distribution	p.d.f.	Mean	Variance
Normal	$\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], -\infty < x < \infty$	μ	σ^2
Gamma	$\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-x/\beta}, 0 \le x < \infty$	$\alpha\beta$	$\alpha\beta^2$
Exponential ($\alpha = 1$)	$\frac{1}{\beta}e^{-x/\beta}, 0 \le x < \infty$	β	β^2
Chi-square ($\alpha = r/2$; $\beta = 2$)	$\frac{1}{2^{r/2}\Gamma(r/2)}x^{(r/2)-1}e^{-x/2}, 0 \le x < \infty$	r	$2r$
Weibull	$\frac{\alpha x^{\alpha-1}}{\beta^\alpha}\exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right], 0 \le x < \infty$	$\beta\Gamma\left(\frac{1}{\alpha}+1\right)$	$\beta^2\left\{\Gamma\left(\frac{2}{\alpha}+1\right) - \left[\Gamma\left(\frac{1}{\alpha}+1\right)\right]^2\right\}$
Lognormal	$\frac{1}{\sqrt{2\pi}\,\sigma y}\exp\left[-\frac{(\ln y-\mu)^2}{2\sigma^2}\right], 0 \le y < \infty$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

Functions in Excel

You can use Excel's **Function Wizard** to implement the exponential distribution and the normal distribution. Excel can give you both the value of the probability density function, *f(x)*, and the value of the cumulative probability distribution, *F(x)*. Just make the last argument of the function **TRUE** if you want *F(x)* and **FALSE** if you want *f(x)*. If you don't want to use the function wizard, you can simply type the functions into a cell just like any other function:

Exponential Distribution	***f(x)*** **= EXPONDIST(*x, lambda, FALSE*)**
	F(x) **= EXPONDIST(*x, lambda, TRUE*)**
Normal Distribution	***f(x)*** **= NORMDIST(*x, mean, stdev, FALSE*)**
	F(x) **= NORMDIST(*x, mean, stdev, TRUE*)**

Excel also lets you implement the cumulative standard normal distribution. The function has just one argument, which is *z*:

Standard Normal Distribution	***F(z)*** **= NORMSDIST(*z*)**

Functions in Excel

Many times, you need to use the cumulative normal distribution *backwards*. For example, to determine the 95th percentile value of X, you have to go into the body of the Cumulative Standard Normal Table, find the value 0.950, figure out what value of Z corresponds to that entry, and convert that value into X. This is called *inversion* and Excel provides functions for solving *inverse* problems involving both the normal distribution and the standard normal distribution:

Normal Distribution **X = NORMINV(*probability, mean, stdev*)**

Standard Normal Distribution **Z = NORMSINV(*probability*)**

The former return the value of X corresponding to the input probability and the latter returns the value of Z corresponding to the input probability.